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A SIMPLE NUMERICAL TRANSMISSION MODEL FOR GENERAL LCD-CONFIGURATIONS

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Abstract

The transmission of a LCD for normal incident light can be simply calculated by a clever use of the Jones calculus. Several cases are treated in different ways. The results for a few examples are compared with the more general simulations according to the 4 × 4 Berreman method, which also takes internal reflections into account. Our fast optical calculations are useful when predicted molecular distributions should be confronted with experimental optical spectra.

1 Introduction

For the simulation of optical phenomena in liquid crystal displays, one often prefers fast algorithms without detailed information above correct but complicated calculations. Especially when the optical transmission is only a part of the simulation output, computer time should be saved. The simulation of the impact of different driving schemes on the molecular switching and the fluctuating transmission is just one example. In this article we present a decent, fast method for optical calculations and we compare the results with a more sophisticated algorithm.

We consider liquid crystal structures where the orientation of the molecules depends on just one coordinate (z). The z-axis is perpendicular to the glass plates. We use the uniaxial approach with an index of refraction $n_{\|}$ along the longitudinal

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molecular axis (the principal axis) and $n_\perp$ along the two other mutually orthogonal directions. The orientation of the indicatrix at an arbitrary position in the liquid crystal layer can be described with two angles γ and ζ as depicted in figure 1. Both angles are functions of z.

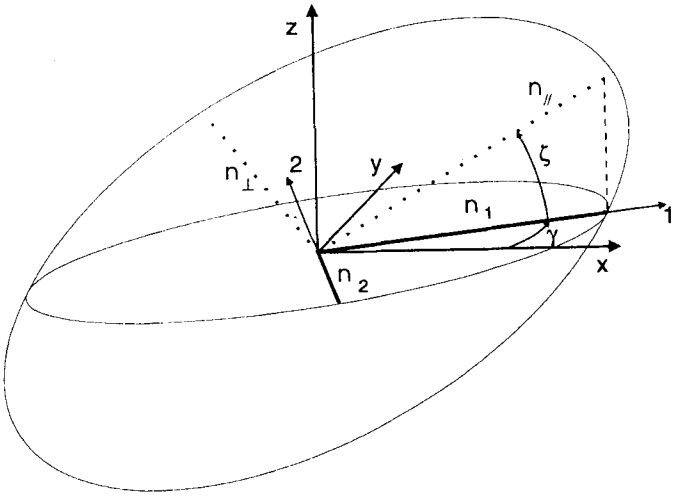


Figure 1: The orientation of the indicatrix, the ellipsoid with axes proportional to the refraction indices. Here the example of a uniaxial material is used, for which two of the three axes are identical.

For the study of the evolution of the polarization through the liquid crystal layers, we need the values n_1 and n_2, respectively along the long and short axis of the horizontal elliptic cross-section. They are related to $n_{||}$ and $n_\perp$ according to the following equations:

$$\begin{aligned} n_1 &= \frac{n_{||} \cdot n_\perp}{\sqrt{n_\perp^2 \cos^2\zeta + n_{||}^2 \sin^2\zeta}} \\ n_2 &= n_\perp \end{aligned} \tag{1}$$

For a horizontal molecule, n_1 and n_2 are equal to $n_{||}$ and $n_\perp$. The effective birefringence for light propagating along the z-axis is given by

$$\Delta n = n_1 - n_2 \simeq (n_{||} - n_\perp)\cos^2\zeta \tag{2}$$

The approximation is valid if $\frac{n_{||}^2 - n_\perp^2}{n_{||}^2}\cos^2\zeta \ll 1$.

Moreover we take account of the dispersion i.e. variation of the refractive indices with the wavelength λ. We use the analytical model [1]

$$n_{||} - n_\perp = G\frac{\lambda^2\lambda_0^2}{\lambda^2 - \lambda_0^2} \tag{3}$$

G and λ_0 are material constants. Numerical values can be found in [2].

2 The Jones calculus

The electric field vector of a plane monochromatic wave propagating along the z-axis can be expressed as follows

$$\begin{aligned}\vec{E}(z) &= E_x e^{j(\Phi_x - kz)}\,\vec{1}_x + E_y e^{j(\Phi_y - kz)}\,\vec{1}_y \\ &= [\frac{1}{\sqrt{2}}(E_x e^{j(\Phi_x - kz)} + jE_y e^{j(\Phi_y - kz)})]\,[\frac{1}{\sqrt{2}}(\vec{1}_x - j\,\vec{1}_y)] + \\ &\quad [\frac{1}{\sqrt{2}}(E_x e^{j(\Phi_x - kz)} - jE_y e^{j(\Phi_y - kz)})]\,[\frac{1}{\sqrt{2}}(\vec{1}_x + j\,\vec{1}_y)] \\ &= E_L e^{j(\Phi_L - kz)}\,\vec{1}_L + E_R e^{j(\Phi_R - kz)}\,\vec{1}_R\end{aligned}$$

The first line shows the decomposition in the usual basis of linear polarizations along the x- and y-axis. Through a complex linear transformation one easily gets the decomposition with respect to the basis of left- and right-circular light. Whatever basis one takes, the vector with the two components contains all necessary information about the electric field. It is the so-called Jones vector [3] with respect to the considered basis.

For a linear basis we have

$$\mathcal{E} = \begin{pmatrix} E_x e^{j(\Phi_x - kz)} \\ E_y e^{j(\Phi_y - kz)} \end{pmatrix}$$

for example $\begin{pmatrix} c\cos\theta \\ c\sin\theta \end{pmatrix}$, with c a complex number denotes linearly polarized light, for which the polarization direction makes an angle θ with the x-axis.

For a circular basis we have

$$\mathcal{E}^{(c)} = \begin{pmatrix} E_L e^{j(\Phi_L - kz)} \\ E_R e^{j(\Phi_R - kz)} \end{pmatrix}$$

If not specified explicitly, we are working with the linearly polarized basis states.

The intensity of a beam is given by $\mid E_x \mid^2 + \mid E_y \mid^2 = \mid E_L \mid^2 + \mid E_R \mid^2 = \mathcal{E}^\dagger \cdot \mathcal{E}$. The evolution of the Jones vector through an optical system can be studied with matrix algebra. We distinguish two cases:

1. We consider an optical component. The relation between the incoming beam (with Jones vector $\mathcal{E}$) and the beam that leaves the component (with Jones vector $\mathcal{E}^\circ$) is given by

$$\mathcal{E}^\circ = \bar{M} \cdot \mathcal{E} \qquad \bar{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{pmatrix} \tag{4}$$

$\bar{M}$ is a 2×2 matrix of which the elements depend on the specific optical properties, such as absorption and optical birefringence. For optical components without absorption or light generation, $\bar{M}$ is a unitary matrix, i.e. $\bar{M} \cdot \bar{M}^\dagger = \bar{1}$.

2. For continuously varying media, the evolution of the vector $\mathcal{E}(z)$ is described with a differential equation:

$$\frac{d\mathcal{E}}{dz} = \bar{N} \cdot \mathcal{E}(z) \qquad \bar{N} = \begin{pmatrix} N_{11} & N_{12} \\ N_{12} & N_{22} \end{pmatrix} \tag{5}$$

Of course, there is a connection between the differential matrix $\bar{N}(z)$ and the integrated $\bar{M}(z)$ that links the beam at $z = 0$ and the beam at z:

$$\frac{d\bar{M}}{dz} = \bar{N} \cdot \bar{M} \tag{6}$$

A rotation of the optical component over an angle ω (positive from x- to y-axis) can be easily described in the Jones calculus (with linearly polarized basis states). The matrices $\bar{M}^\star$ and $\bar{N}^\star$ for the rotated component relate to the non-rotated matrices $\bar{M}$ and $\bar{N}$ according to the following transformation rules:

$$\bar{M}^\star = \bar{S}(\omega) \cdot \bar{M} \cdot \bar{S}(-\omega) \tag{7}$$

$$\bar{N}^\star = \bar{S}(\omega) \cdot \bar{N} \cdot \bar{S}(-\omega) \tag{8}$$

with

$$\bar{S}(\omega) = \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} \tag{9}$$

3 Calculation of the Jones matrix

3.1 Polarizers

A polarizer along the x-axis does not influence light that is linearly polarized in the same direction, but extinguishes light that is linearly polarized in the orthogonal direction. One easily verifies that the $\bar{M}$-matrix, which we denote $\bar{P}$ in this case, looks as follows:

$$\bar{P} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{10}$$

The matrix $\bar{P}^\star$ for a polarizer that makes an angle θ with the x-axis, can be calculated with equation (7) with $\omega = \theta$.

3.2 Homogeneous uniaxial media

For the transmission of a homogeneous slab of uniaxial material, we refer to figure 1. In this case γ and ζ are constant. The thickness of the slab is d. Linearly polarized light with its direction of polarization along axis 1 propagates in a way that corresponds with the refraction index n_1. Similarly, if the polarization direction is along axis 2, the propagation is described with n_2.

Let us first describe the Jones calculus in the *1,2*-frame. For the $\bar{N}$-matrix we have:

$$\bar{N} = -j\frac{2\pi}{\lambda}\begin{pmatrix} n_1 & 0 \\ 0 & n_2 \end{pmatrix} \tag{11}$$

λ is the wavelength of the considered light. With equation (6) one can calculate the matrix $\bar{M}$ which we give the name $\bar{H}$ in this case:

$$\bar{H} = \begin{pmatrix} e^{-j2\pi n_1 \frac{d}{\lambda}} & 0 \\ 0 & e^{-j2\pi n_2 \frac{d}{\lambda}} \end{pmatrix} \tag{12}$$

Since the *1*-axis actually makes an angle γ with the x-axis, we can calculate the Jones matrix $\bar{H}^\star$ in the xy-frame by application of the transformation rule (7) with $\omega = \gamma$.

3.3 Uniformly twisted layer

In case the tilt ζ is constant, and the twist γ is a linear function of z, we are dealing with a uniformly twisted or helicoidal structure. With p the pitch of the helix, we first define the following quantities:

$$\begin{aligned} \alpha &= \frac{2\pi}{p} \\ \Phi &= \alpha d \\ g &= \frac{\pi \Delta n}{\lambda} \\ u &= \frac{g}{\alpha} \\ &= \frac{\pi \Delta n}{\Phi}\frac{d}{\lambda} \qquad (13) \\ \beta &= \alpha\sqrt{1+u^2} \qquad (14) \end{aligned}$$

Φ is the total twist across the LC-layer.

First, we consider a uniformly twisted layer with the easiest orientation: the molecular *1*-axis at the entrance side is parallel with the x-axis. Starting with the differential matrix $\bar{N}$ from equation (11), one can construct the $\bar{M}$-matrix of this twisted crystal, which we call $\bar{T}$ in this case. According to Jones [3]

$$\bar{T} = \bar{S}(\alpha z)exp[(\bar{N}_0 - \alpha\bar{S}(\frac{\pi}{2})).z] \tag{15}$$

with $\bar{S}$ defined as in equation (9). Finally we get (with $\overline{a+jb} = a - jb$)

$$\begin{aligned} T_{11} = \overline{T_{22}} &= \cos\alpha z\ \cos\beta z + \frac{\alpha}{\beta}\sin\alpha z\ \sin\beta z + j\frac{g}{\beta}\cos\alpha z\ \sin\beta z \\ T_{12} = -\overline{T_{21}} &= \frac{\alpha}{\beta}\cos\alpha z\ \sin\beta z - \sin\alpha z\ \cos\beta z + j\frac{g}{\beta}\sin\alpha z\ \sin\beta z \end{aligned} \tag{16}$$

For a more general configuration, where the *1*-axis at the entrance side makes an angle γ with the x-axis, one should use equation (7) with $\omega = \gamma$ for the calculation of the Jones matrix $\bar{T}^\star$.

3.4 General distribution

For general functions $\gamma(z)$ and $\zeta(z)$, the calculation of the Jones-matrix $\bar{M}$ is not simple anymore. One has to solve the set of four differential equations (6). In most of the cases, this can only be done numerically.

A faster numerical method is the following. We treat the liquid crystal layer as a stratified medium, i.e. a stack of very thin slabs S_i $(i = 1 \rightarrow n)$. In each slab, the tilt angle ζ is considered to be constant. For the twist angle γ we can choose between two approximations: either we use a constant γ within each slab or we take a linear function. This means that we treat each slab either as homogeneous or as uniformly twisted. The resulting approximations are illustrated in figure 2.

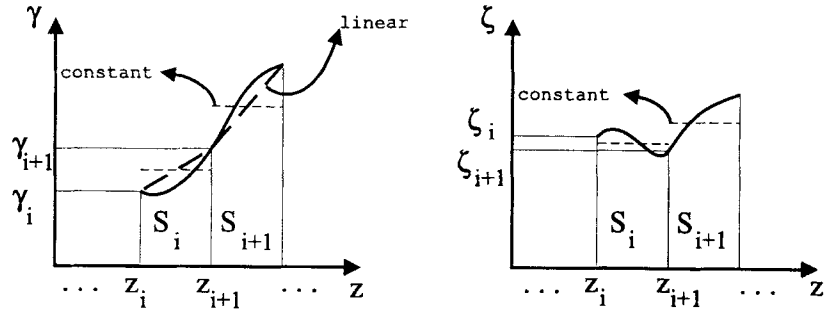


Figure 2: Within each slab S_i, the twist γ can be approximated either by a constant or a linear function. For the tilt-evolution ζ we only use the constant approximation.

For each of the slabs one can define the Jones matrix $\bar{H}^\star$ (see section 3.2) or $\bar{T}^\star$ (see section 3.3). The Jones-matrix of the total LC-layer can be easily calculated by matrix multiplication

$$\bar{M} = \bar{H}_n^\star \cdot \bar{H}_{n-1}^\star \cdots \bar{H}_2^\star \cdot \bar{H}_1^\star \tag{17}$$

or

$$\bar{M} = \bar{T}_n^\star \cdot \bar{T}_{n-1}^\star \cdots \bar{T}_2^\star \cdot \bar{T}_1^\star \tag{18}$$

4 The χ-calculus

4.1 Description

If we are dealing with non-absorbing material, we only need to study the evolution of the ellipse of polarization of the light wave. The Jones vector has information on the total amplitude and absolute phase of the wave, as well as the shape of the polarization ellipse. Information on the polarization ellipse can be extracted from the Jones vector by taking the ratio of its two components. This ratio χ

is one single complex variable from which both the azimuth θ and the ellipticity $e = \tan \epsilon$ can be obtained (see figure 3). Azzam and Bashara [4] developed this useful technique.

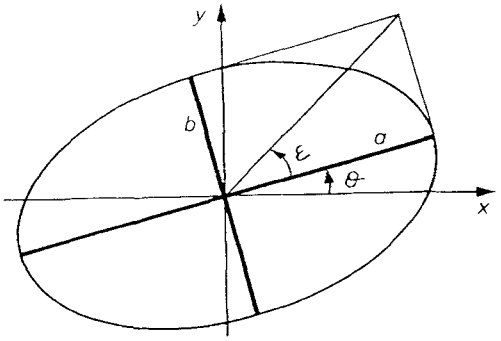


Figure 3: The shape and the orientation of the polarization ellipse (with axes a en b) are defined by the angles ϵ and θ.

Let us consider the differential equations (5)

$$\begin{cases} \frac{d\mathcal{E}_1}{dz} &= N_{11}\,\mathcal{E}_1 + N_{12}\,\mathcal{E}_2 \\ \frac{d\mathcal{E}_2}{dz} &= N_{21}\,\mathcal{E}_1 + N_{22}\,\mathcal{E}_2 \end{cases} \tag{19}$$

Starting with these equations we can derive one single differential equation for the ratio $\chi = \frac{\mathcal{E}_2}{\mathcal{E}_1}$

$$\frac{d\chi}{dz} = -N_{12}\chi^2 + (N_{22} - N_{11})\,\chi + N_{21} \tag{20}$$

This differential equation has a solution $\chi(\chi_0, z)$, where χ_0 is the starting value at $z = 0$. The solution can be explicitly found for those configurations that could be described by explicit Jones matrices: the homogeneous and the uniformly twisted case.

4.2 Linearly polarized basis states

Especially for the study of homogeneous layers, it is convenient to consider two orthogonal linear polarizations as basis states ($\chi = \frac{E_y}{E_x}$). In this case, the differential equation (20) has constant coefficients. A general solution can be found, see [4].

If we choose the x- and y-directions parallel to the principal axes of birefringence, the $\bar{N}$-matrix has the simple form of equation (11). The solution of equation (20) then looks as follows:

$$\chi(\chi_0, z) = e^{j2\pi(n_1 - n_2)\frac{z}{\lambda}}\,\chi_0 \tag{21}$$

For linearly polarized incoming light, $\chi_0 = \tan \theta_0$, where θ_0 is the angle between the polarization direction and the x-axis. The azimuth θ and the ellipticity $e = \tan \epsilon$

of the polarization ellipse can be derived from (21) through the following relations:

$$\begin{aligned} \tan 2\theta &= 2\Re(\chi)/(1- \mid \chi \mid^2) \\ \sin 2\epsilon &= 2\Im(\chi)/(1+ \mid \chi \mid^2) \end{aligned} \tag{22}$$

4.3 Circularly polarized basis states

For uniformly twisted layers (of which the homogeneous layer is a limiting case), we take left- and right-circularly polarized basis states.

$$\chi = \frac{E_R}{E_L} = \frac{E_x - jE_y}{E_x + jE_y}$$

The axes x and y are chosen parallel to the principal axes of birefringence of the molecules at $z = 0$. It is important to remember this choice. The $\bar{N}$-matrix, which we should use for the twisted structure is (see also [4])

$$\bar{N}^{(c)} = \frac{\pi}{\lambda}(n_2 - n_1) \begin{pmatrix} 0 & je^{j2\alpha z} \\ je^{-j2\alpha z} & 0 \end{pmatrix} \tag{23}$$

α is defined as in section 3.3. Azzam and Bashara [4] showed a technique to solve equation (20) in this case. Our reference frames are slightly different from theirs. In reference [5], one of us derived the following propagation formulas:

$$\begin{aligned} \chi(\chi_0, z) &= \chi^*(\chi_0, z)e^{i2\alpha z} \\ \chi^*(\chi_0, z) &= \frac{(\beta - i\,\alpha \tan\beta z)\chi_0 + i\,g \tan\beta z}{(i\,g \tan\beta z)\chi_0 + (\beta + i\,\alpha \tan\beta z)} \end{aligned} \tag{24}$$

For the definition of β and g, see section 3.3. In the homogeneous case we have $\alpha = 0$, $\Phi = 0$, $u \to \infty$ and $\beta = g$.

$\chi(\chi_0, z)$ and $\chi^*(\chi_0, z)$ both have the information of the polarization ellipse. The difference is that $\chi(\chi_0, z)$ uses the molecular axes at the entrance side ($z = 0$) as reference frame, while χ^* is related to the molecular axes at the spot z itself.

The relations with the classical parameters of the polarization ellipse are:

$$\begin{aligned} \theta &= \frac{1}{2} Arg(\chi) \\ \tan\epsilon &= \frac{1- \mid \chi \mid}{1+ \mid \chi \mid} \end{aligned} \tag{25}$$

One can write down the same relations for χ^*. For the ellipticity this will not make any difference, the angle θ^* will be αz smaller, which confirms the distinction between χ and χ^*.

The evolution of χ in the complex plane can be related with a stereographic projection of the Poincaré sphere. This is clearly illustrated in reference [5], for several liquid crystal configurations.

4.4 The general distribution

For general functions $\gamma(z)$ and $\beta(z)$, we can use the χ-calculus with a technique similar to section 3.4. The total liquid crystal layer is split into slabs S_i ($i = 1 \rightarrow n$). In each slab we take a constant ζ, from which we calculate the appropriate birefringence values Δn, β and g. As in section 3.4, we can make either a constant or linear approximation for the twist γ. In both cases, we will a circular bases (formulae 24). The χ-value at the exit of one slab becomes the starting value for the next slab. However, the incoming χ_0 should always be referred to the appropriate axes frame: the principal axes of birefringence of the molecules at the entrance of the slab.

Let γ_i be the molecular twist at the beginning of the slab S_i (between z_i and z_{i+1}, see figure 2).

If we use the approximation of homogeneous slabs ($\alpha = 0$), we should execute a rotation of χ after each slab. If χ_i denotes the χ-value at the end of slab S_i and d_i the thickness of slab S_i, we therefore use equation (21) in the following scheme:

$$\begin{aligned}
\chi_1 &= \chi(\chi_0, d_1) \\
\chi_2 &= \chi(\chi_1 \cdot e^{-2j(\gamma_2-\gamma_1)}, d_2) \\
&\vdots \\
\chi_{i+1} &= \chi(\chi_i \cdot e^{-2j(\gamma_{i+1}-\gamma_i)}, d_{i+1}) \\
&\vdots \\
\chi_n &= \chi(\chi_{n-1} \cdot e^{-2j(\gamma_n-\gamma_{n-1})}, d_n) \\
\chi_{final} &= \chi_n \cdot e^{2j(\gamma_n-\gamma_1)}
\end{aligned} \tag{26}$$

The factor 2 in the exponents is related with equation (25), which shows that only half of the argument of χ has a physical meaning. With the last rotation we express the final χ-value in the reference frame that coincides with the molecular axes at the entrance side of the total LC-layer, which should also be used as reference for χ_0.

If we take the uniformly twisted slabs, we can use formulae (24) in a clever way: we don't need to bother about intermediate rotations. With equations (24), we get the following scheme:

$$\begin{aligned}
\chi_1^* &= \chi^*(\chi_0, d_1) \\
\chi_2^* &= \chi^*(\chi_1^*, d_2) \\
&\vdots \\
\chi_{i+1}^* &= \chi^*(\chi_i^*, d_i) \\
&\vdots \\
\chi_n^* &= \chi^*(\chi_{n-1}^*, d_n) \\
\chi_{final} &= \chi_n^* \cdot e^{2j\Phi}
\end{aligned} \tag{27}$$

Again the last rotation (with the total LC-twist Φ) brings χ in the initial reference frame.

For the transmission through the analyzer, one should look for the projection of the elliptical polarization on the analyzer's direction. It is given by

$$T(\lambda) = \frac{1}{2}[1 + \cos 2\epsilon \cos 2\psi]$$

where ψ is the angle between the analyzer and the ellipse's long axes. If both are referred with respect to the initial reference frame, we can write:
$\psi = \gamma_a - \frac{1}{2} Arg(\chi_{final})$.

5 Examples

5.1 Homogeneous slab between polarizers

If one considers a homogeneous liquid crystal layer or a solid birefringent crystal between two crossed polarizers, one can easily calculate the transmission for any incoming linearly polarized beam. We will use the formalism with the Jones matrices from section 3.2. We take the polarizer along the x-axis and the analyzer along the y-axis. The molecular direction makes an angle γ with the polarizer and there is an angle θ between the polarization direction of the incoming beam and the polarizer.

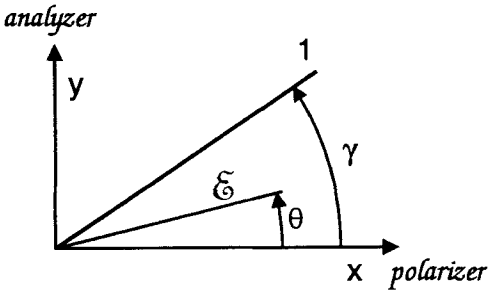


Figure 4: For the study of the transmission through a homogeneous slab we choose the x-axis along the polarizer. The angles for the other elements are: θ for the incoming wave vector $\mathcal{E}$, γ for the molecular 1-axis and $\frac{\pi}{2}$ for the analyzer.

With the configuration as illustrated in figure 4, we get

$$\mathcal{E} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$\mathcal{E}^{\circ} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix} \cdot \begin{pmatrix} e^{-j2\pi n_1 \frac{d}{\lambda}} & 0 \\ 0 & e^{-j2\pi n_2 \frac{d}{\lambda}} \end{pmatrix} \cdot$$

$$\begin{pmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \sin\gamma\ \cos\gamma\ (e^{-j2\pi n_1 \frac{d}{\lambda}} - e^{-j2\pi n_2 \frac{d}{\lambda}})\ \cos\theta \end{pmatrix}$$

The relative transmission is then given by

$$\begin{aligned} T &= \frac{\mathcal{E}^{\circ\dagger} \cdot \mathcal{E}^{\circ}}{\mathcal{E}^{\dagger} \cdot \mathcal{E}} \\ &= \sin^2 2\gamma\ \sin^2(\pi \Delta n \frac{d}{\lambda})\ \cos^2\theta \end{aligned} \tag{28}$$

This well-known result (28) can be derived in other ways: by explicit propagation formulae for the ordinary and extra-ordinary wave [6] or by the χ-calculus [5].

5.2 Twisted nematic liquid crystals

For the transmission of TN-(twisted nematic) or STN-(supertwisted nematic)LCD's in all kinds of configurations, we refer to reference [5], where one of us used the χ-calculus to study the propagation of polarized light in liquid crystal structures. A concise résumé can be found in the next formula, already published in another article by some of us [2]

$$\begin{aligned} T(\lambda) &= \frac{1}{2}\{1 + [(\frac{g^2}{\beta^2} + \frac{\alpha^2}{\beta^2}\cos 2\beta d)\ \cos 2\gamma_p + \frac{\alpha}{\beta}\sin 2\beta d\ \sin 2\gamma_p]\cos 2(\gamma_a - \Phi) \\ &\quad + [\cos 2\beta d\ \sin 2\gamma_p - \frac{\alpha}{\beta}\sin 2\beta d\ \cos 2\gamma_p]\sin 2(\gamma_a - \Phi)\} \end{aligned}$$

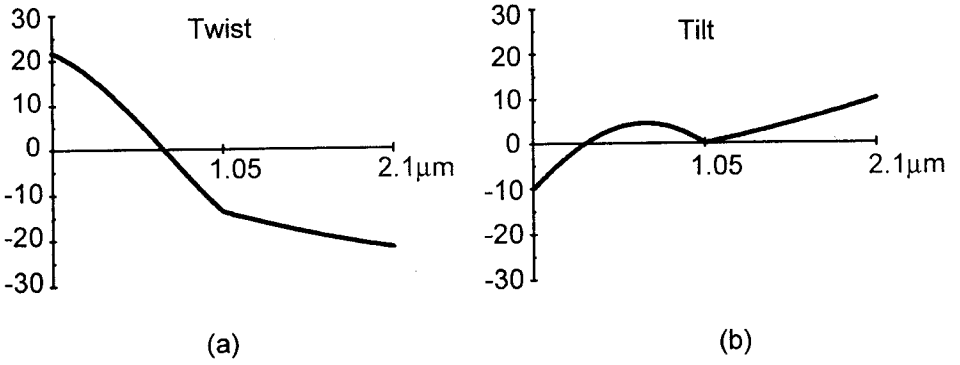
The transmission at a certain wavelength $T(\lambda)$ depends on the four parameters: (1) Φ (total twist), (2) $\Delta n\ d$ (retardance), (3) γ_a (position analyzer), and (4) γ_p (position polarizer). γ_a and γ_p are defined with respect to the molecular position at the side of the light entrance.

α, β, g and u were defined in equations (14).

5.3 Laminar approximations

For specific defect structures in ferroelectric liquid crystal displays, a stack of homogeneous sublayers is an efficient approach [7]. In this reference we used the χ-calculus according to scheme (26).

In figure 5, we present some results for a general twist and tilt distribution, also in ferroelectric material. The results can be obtained both with the matrix multiplication (equation 17) or with the χ-calculus (equation 26).



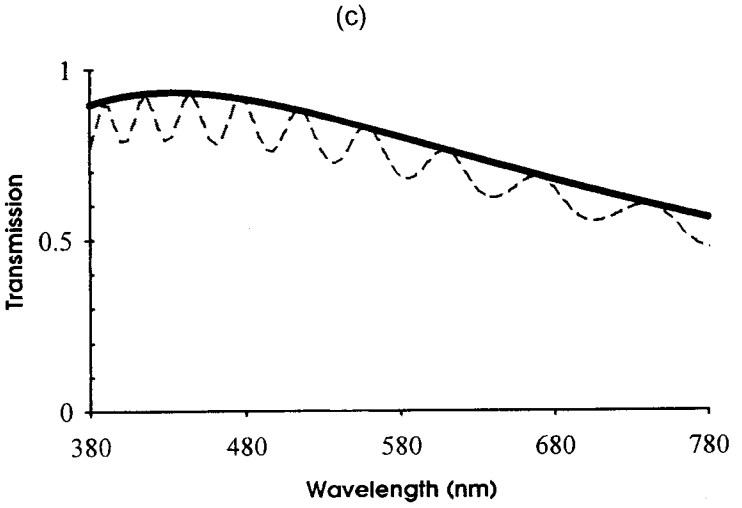


Figure 5: With the twist-distribution as in figure (a) and the tilt-evolution as in figure (b) we calculated the optical transmission (c). Both the result from the Jones calculus (solid line) and the spectrum calculated by a 4×4 matrix method (dashed line) are depicted.

In figure 5c, the transmission spectrum is compared with the oscillating spectrum that results from our 4×4 matrix calculations according to Berreman [8]. These full calculations have been implemented in a simulation program [9]. They take account for the interference of reflected light waves at the liquid crystal-glass interface. The maxima of the detailed spectrum lie on the full curve, obtained from the Jones calculus. The reflection of light causes a decrease of the transmission that depends on the wavelength. This result seems more reasonable to us than data in literature [10] where the oscillating curve exceeds the Jones curve.

5.4 Confrontation with experimental results

One of us used the presented calculation technique to design a switchable color filter with two 180° supertwisted nematic cells as depicted in figure 6.

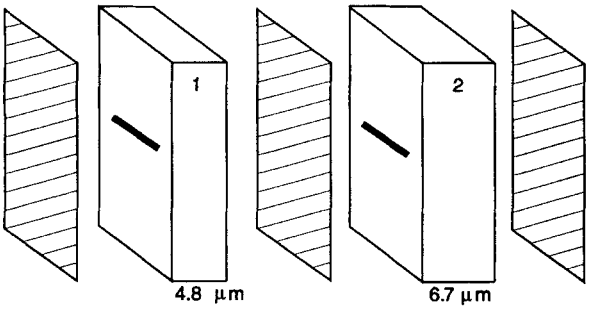


Figure 6: The design of a switchable color filter. Three polarizers are oriented in a parallel way (making an angle of 45° with the horizontal). The two 180° supertwisted nematic cells are put in such a way that the molecules at the glass plates are horizontal. The first has a thickness of $4.8 \mu m$, the second is $6.7 \mu m$ thick

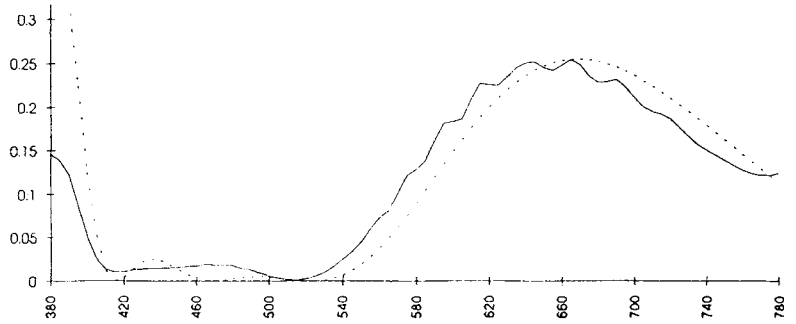


Figure 7: In this figure we compare the measured spectrum (solid line) with the simulated one (dashed line), for one of the switched states.

If a voltage of 4 Volt is applied to the first cell (4.8μm thickness) and the second cell is in short circuit, the transmitted spectrum represents a red color. In figure 7 we compare the measured spectrum with the calculated one. For the calculation, we used a linear $\gamma(z)$-function and a parabolic tilt $\zeta(z)$. This molecular distribution could be predicted by other simulations.

6 Conclusions

We presented fast algorithms for the calculation of the optical transmission through a liquid crystal structure. They are based on the classical Jones calculus. Comparison with more sophisticated simulations shows the quality of our work. We are frequently using the calculation technique in theoretical simulations and optical component design.

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